Calculations or sums as we used to call them…

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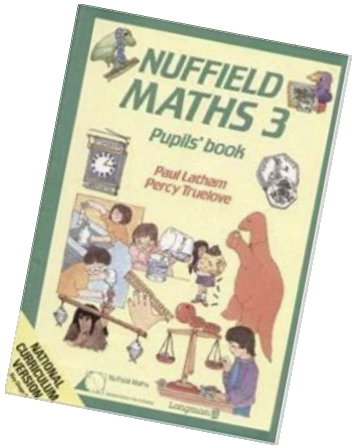
Why is maths taught differently now?

When I was at school I remember the few occasions that the teacher stood at the front of the class and did some maths teaching, it involved teaching rules of how we do things. For example: long multiplication. We didn't question and often we didn't understand why we were "carrying ten" or "borrowing one".



The rest of the time I was taught Maths through a text book and would line up at the teacher's desk to get my work marked once I had finished.

Activities like this were probably okay for the children who 'got' maths, but not for those who struggled or were uninspired by the work on offer.

By the end of year 2, pupils should know the number bonds to 20 and be precise in using and understanding place value. An emphasis on practice at this early stage will aid fluency.

By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work.

By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.

Without secure mental calculation children cannot move onto written methods of calculation.

Children need to:

- be taught the specific skills needed in calculation;
- understand what they are doing and why;
- enjoy maths;
- journey through education with a solid understanding in maths in order to help them in adult life.

Please remember that each child is an individual and all children develop their mathematical understanding at a different pace.

Vocabulary

Add, adding, addition

Number sentence

Calculation

Sum (only to be used for addition calculations, not subtraction)

Total

altogether

Counting up

Partitioning

Number line

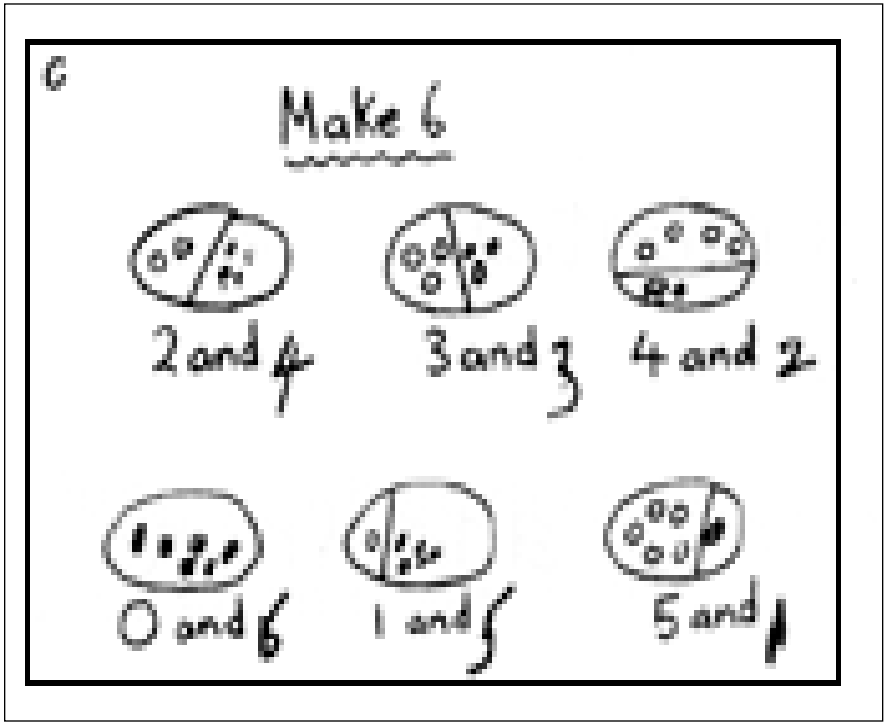
Number bonds

rounding

Addition

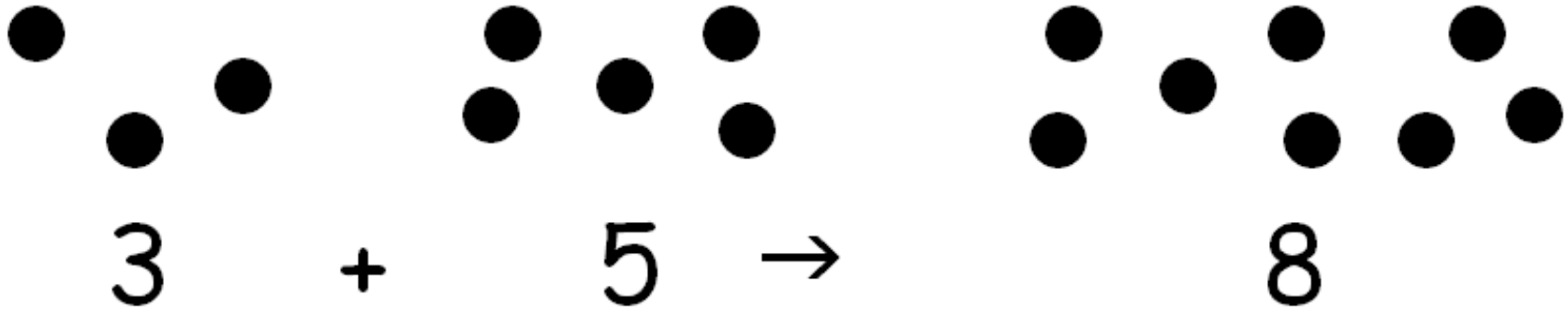
The first stage in written addition

There is a large emphasis on counting and knowing what numbers look like, for example knowing that two counters is the value ‘two’.



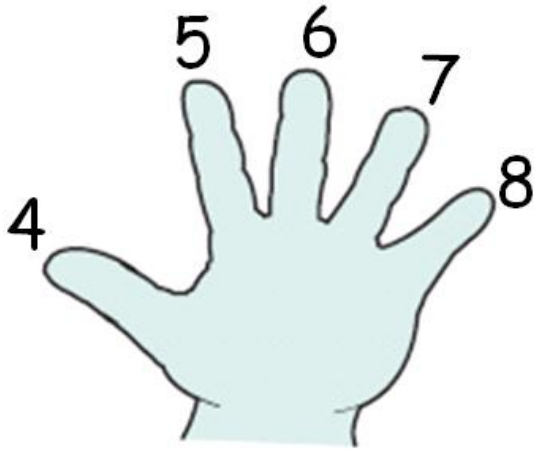
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.

Count all – a child doing 3 + 5 counts out three counters and then five counters and then finds the total by counting all the counters.



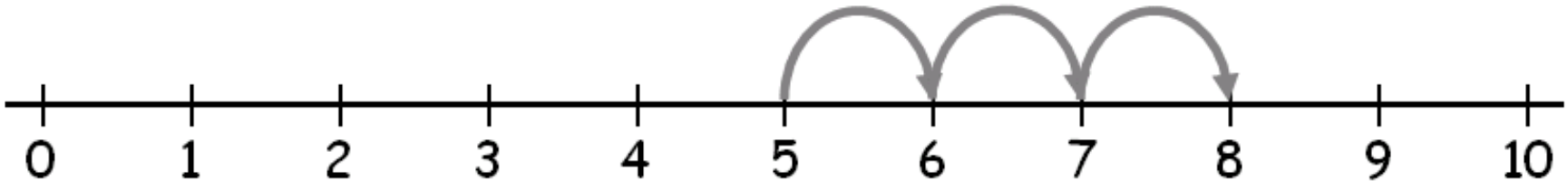
'3'

Count on from the first number – a child finding 3 + 5 counts on from the first number: *'four, five, six, seven, eight'*.



'5'

Count on from the larger number : 3 + 5 a child chooses the larger number, even when it is not the first number, and counts on from there: *'six, seven, eight'*



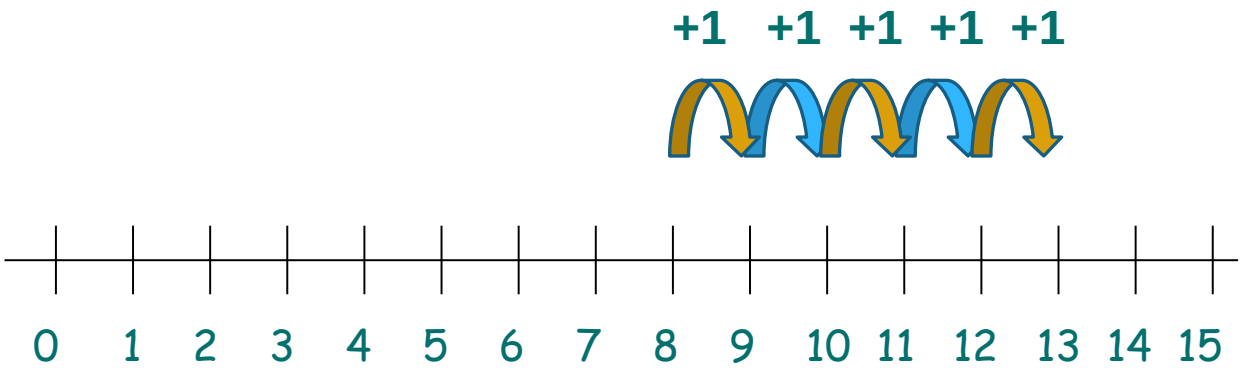
As well as children using their fingers to add on small amounts, a number line will be introduced.

It is more efficient to count on from the larger number because you have less to work out. It also shows children that addition can be done in any order ; it doesn't matter which number you add first, you get the same answer.

The second stage in addition

Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

8 + 5 = 13

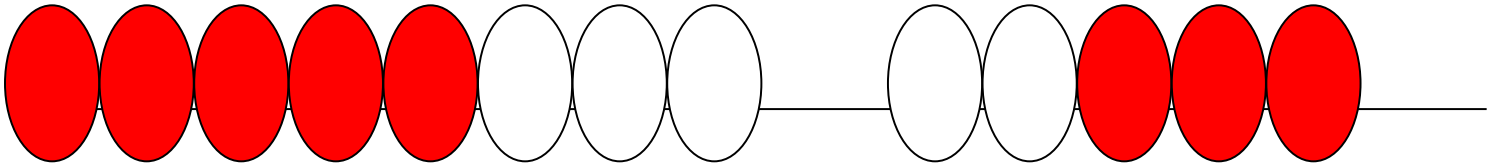


addition including bridging through ten by counting on 2 then counting on 3.

(We call it bridging through ten, when calculations take you over ten).

For example: 8 + 5 =
8 + 2 (takes you to 10)
Then 10 +3 = 13.

This is why it is vital that children know their number bonds to 10 and their single digit facts.

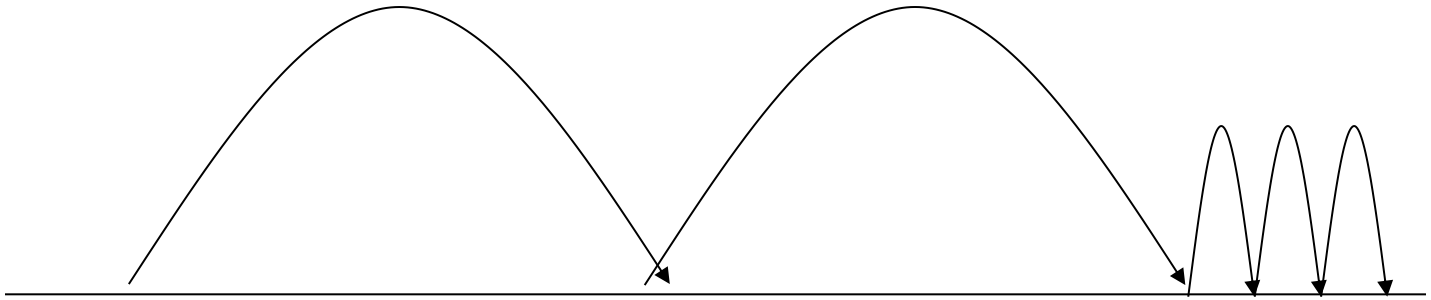


Children will begin to use 'empty number lines' themselves starting with the larger number and counting on.

First counting on in tens and ones.

34 + 23 = 57

Partitioning is important here: knowing that 23 = 10 + 10 +1 +1 +1

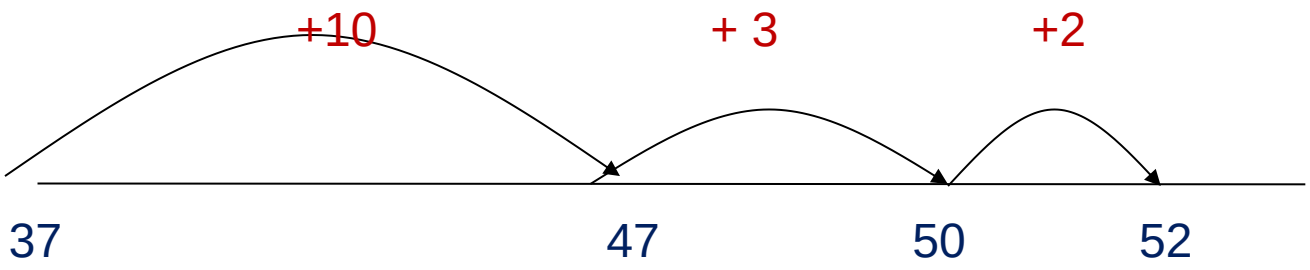


Then helping children to become more efficient by adding the units in one jump (by using the known fact 4 + 3 = 7).

Followed by adding the tens in one jump and the units in one jump.

Bridging through ten can help children become more efficient.

37 + 15 = 52



Notice how the addition calculation is presented horizontally and not vertically in columns.

The third stage in written addition

Children will continue to use empty number lines with increasingly large numbers.

Children use jottings to record mental working out. They begin to use an expanded layout that looks a lot more like the column method, that they will learn to use in upper key stage two.

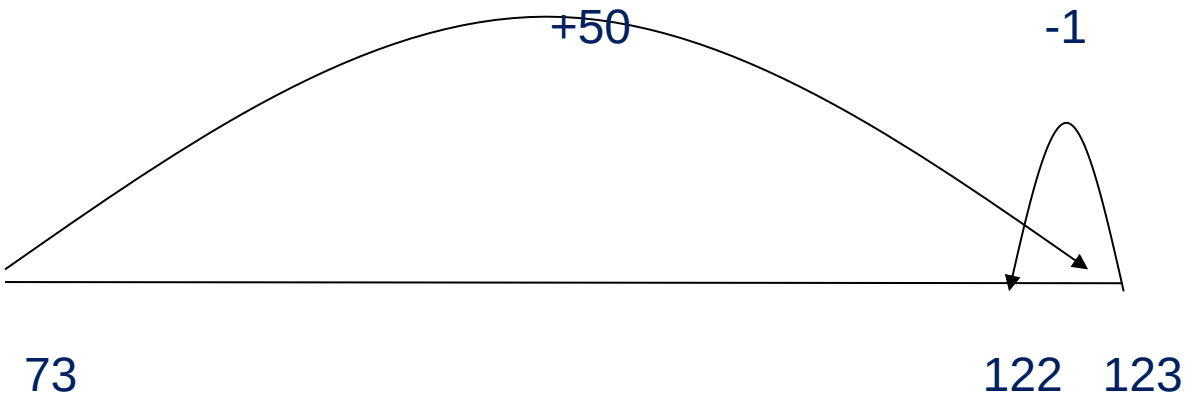
$$\begin{array}{rll} 85 & = 80 + 5 & \\ +\ \underline{46} & \underline{40 + 6} & \\ & 120 + 11 & = 131 \end{array}$$

Partitioning is used here again. Splitting the number up into tens and units.

Children may use their knowledge of rounding to the nearest ten number to help them with addition calculations.

49 + 73 = 122

73 + 50, then -1



The fourth stage in written addition

Using partitioning the children will set out the working out in columns. Adding the units, then the tens and so on...

```
  67
+ 24
  11  ( 7 +  4)
  80  (60 + 20)
  91
```

```
  267
+ 85
  12   ( 7 + 5 )
 140   ( 60 + 80 )
 200   ( 200 + 0 )
 352
```

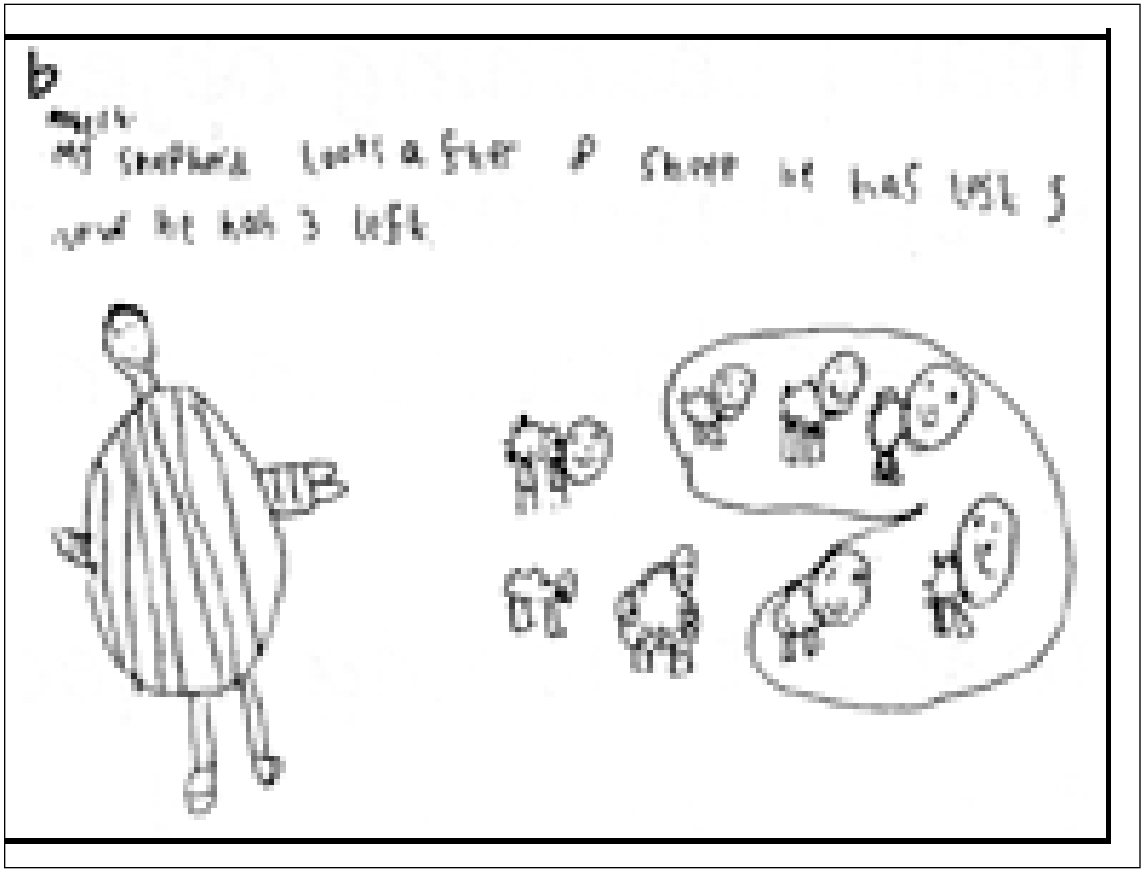
Vocabulary

Subtract, subtracting, subtraction
Number sentence
Calculation
Take away
difference
Counting on, counting back
Partitioning
Number line
Number bonds
Less than/ fewer than

Subtraction

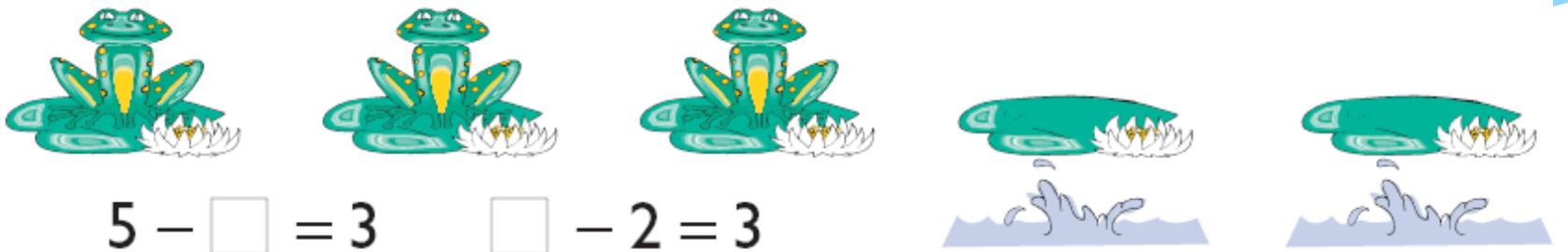
The first stage in written subtraction

Children develop ways of recording calculations using pictures etc.



Counting back – taking away

There were five frogs. Two jumped into the pond. How many were left?



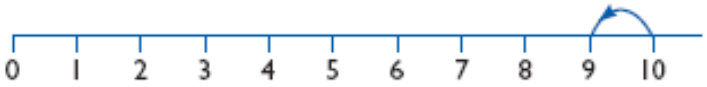
$5 - \square = 3$ $\square - 2 = 3$

1 less than 10

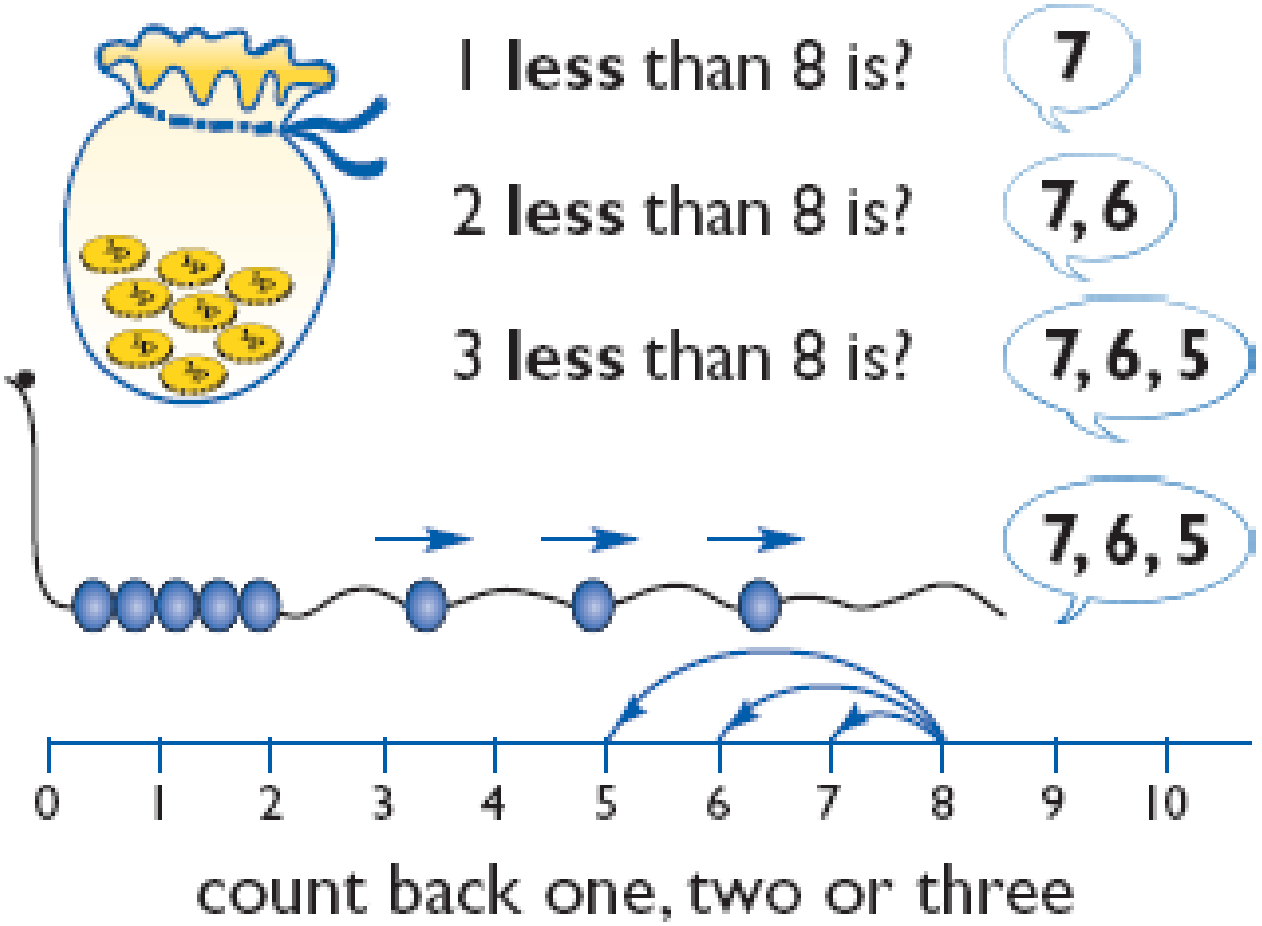
1 **less** than 10 is 9

10 **subtract** 1 equals 9

10 – 1 = 9



A chocolate bar cost 8p. The shopkeeper had a sale and took 3p off. How much does the chocolate bar cost now?

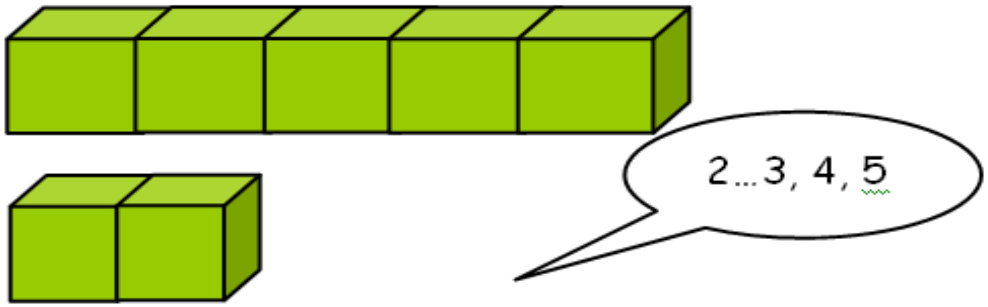


10 and 4 less

With subtraction calculations we are careful not to over use the vocabulary of 'taking away' because some subtraction calculations involve 'finding the difference'. We use the word 'subtract'.

Counting on – finding a difference

Max has 5 cubes. Milly has 2 cubes. How many more cubes does Max have?

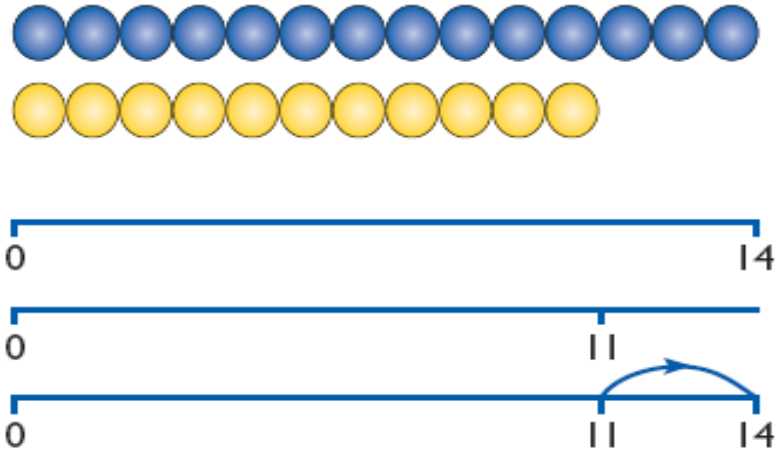


How many less/fewer cubes does Milly have?
What is the difference between?

A teddy costs 50p and doll costs 20p. How much more does the teddy cost?

How much cheaper is the doll?

If my friend is 14 and his sister is 11, how much older is my friend?



The difference between 11 and 14 is 3.

$14 - 11 = 3$

$11 + \square = 14$

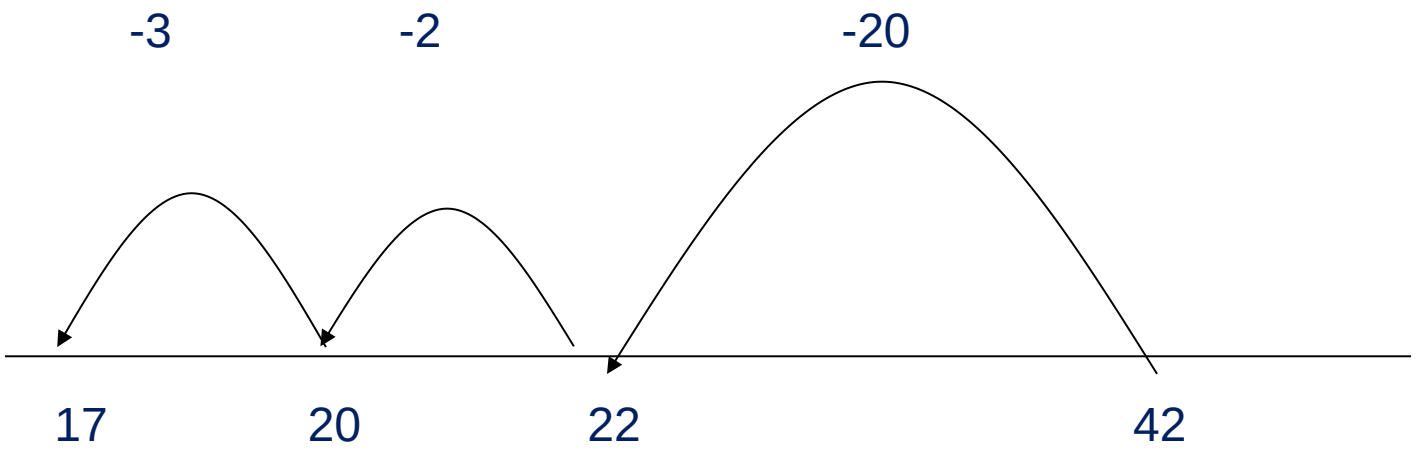
A difference can be found by counting up from the smaller number to the larger number. E.g. 10 – 5 = 5. Count up from 5 to 10 and the difference is 5. A number line may be used for this.

The second stage in written subtraction

Children will begin to use **empty number lines** to support calculations.

Counting back – taking away

42 – 25 = 17



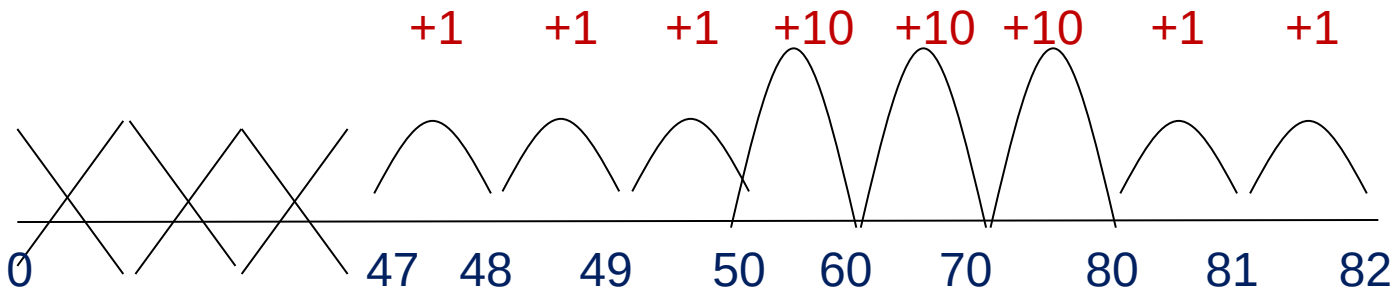
Children may begin by subtracting the tens separately, then the units separately. So the following number line (counting back from 42) would read -10, -10, -2, -3. This number line shows how bridging through ten has been used, where the child splits up the ‘5’ in ‘25’ into ‘2’ and ‘3’. Again single digit facts and number bonds to 10 need to be known.

Counting on – finding a difference

Count up from 47 to 82 in jumps of 10 and jumps of 1.

The number line should still show 0 so children can cross out the section from 0 to the smallest number. They then associate this method with 'taking away'.

82 – 47



(counting in 1s) (counting in 10s) (counting in 1s)

The third stage in written subtraction

Children will continue to use **empty number lines** with increasingly large numbers.

Counting on – finding a difference

Children continue to count on in the context of problem solving
e.g. Two snakes are 56 cm and 83 cm long. What is the difference in their lengths?

Counting back – take away/partitioning

Children develop their use of the empty number line to support their calculations. They **begin to record subtraction calculations vertically** that cannot be easily done mentally. They partition one of the numbers and add or subtract the units, tens and hundreds separately:

267 – 149

267
- 9
258
- 40
218
-100
118

Partitioning and decomposition

$$\begin{array}{rcl} 89 & = & 80 + 9 \\ -\ 57 & & 50 + 7 \\ \hline & & 30 + 2 = 32 \end{array}$$

Initially, the children will be taught using examples that do not need the children to exchange (what you might know as borrowing).

From this the children will begin to exchange.

The calculation should be read as e.g. take 6 from 1.

$$\begin{array}{rcl} 71 & = & = \\ -\ 46 & & \end{array}$$

Step 1

$$\begin{array}{rcl} 70 + 1 \\ -\ 40 + 6 \end{array}$$

Step 2

$$\begin{array}{rcl} {}^{60}\not{7}0 + {}^{1}1 \\ -\ 40 + 6 \\ \hline 20 + 5 = 25 \end{array}$$

Where the numbers involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.

Children should know that units line up under units, tens under tens, and so on.

The fourth stage in written subtraction

Partitioning and decomposition

754 =
- 86

Step 1 700 + 50 + 4
- 80 + 6

Step 2 700 + ~~50~~ (40) + 14 *(adjust from T to U)*
- 80 + 6

Step 3 ~~700~~ (600) + 140 + 14 *(adjust from H to T)*
- 80 + 6
600 + 60 + 8 = 668

This would be recorded by the children as

~~700~~ (600) + ~~50~~ (140) + 14 (1)
- 80 + 6
600 + 60 + 8 = 668

Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc or where a lot of borrowing will be needed, counting on using a number line should be used.

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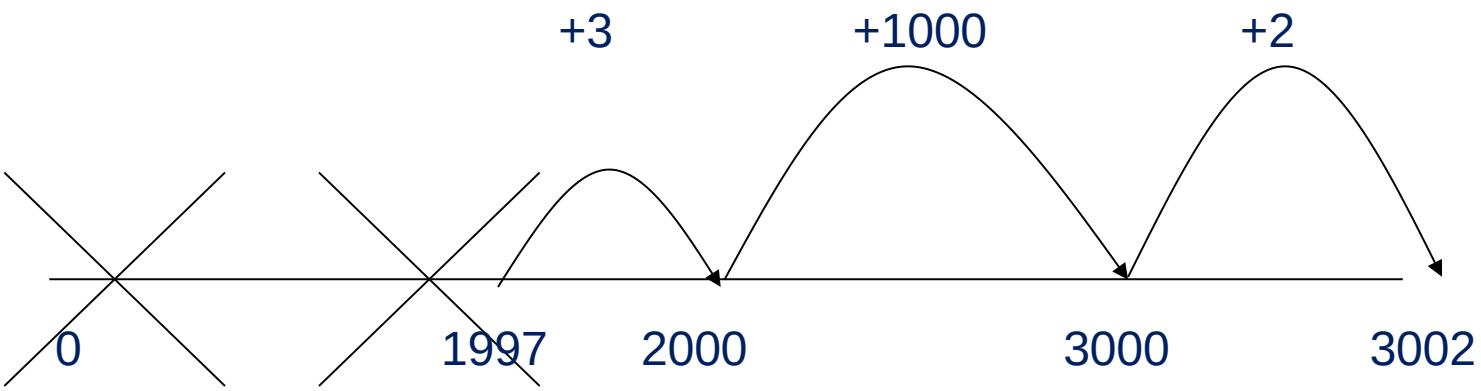


Vocabulary

Multiply, multiplying, multiplication

Number sentence

Calculation

Repeated addition

Partitioning

Number line

Array

Grid method

Lots of

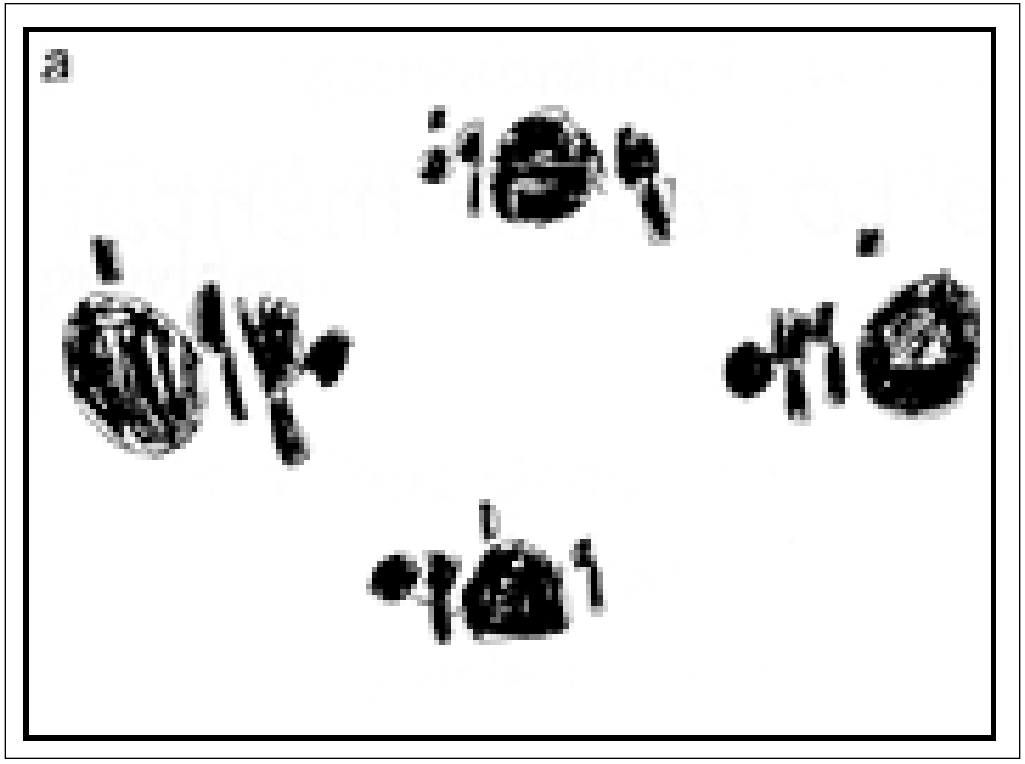
How many times

times

Multiplication

The first stage in written multiplication

Children will experience equal groups of objects and will begin to count in 2s, 10s and 5s. They will work on practical problem solving activities involving equal sets or groups.



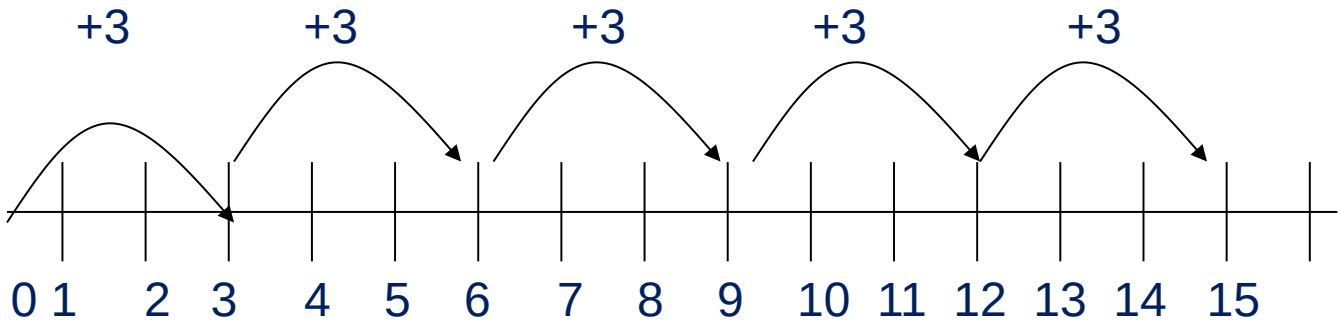
The second stage in written multiplication

Children will develop their understanding of multiplication and use jottings to support calculation:

Repeated addition

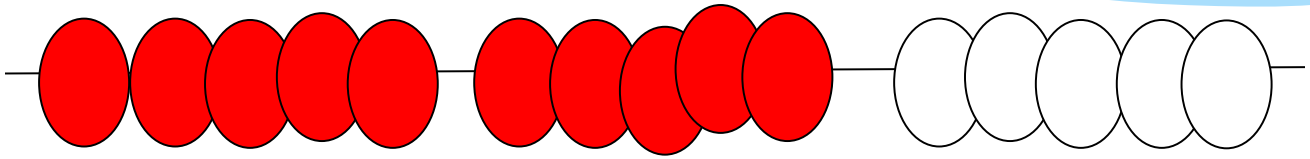
Repeated addition can be shown easily on a number line:

5 x 3 = 3+3+3+3+3 (5 lots of 3)



Show on bead bar or on a number line:

3 x 5 = 5 + 5 + 5

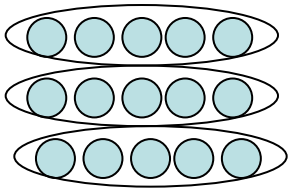


Children need to learn through practice that multiplication can be done in any order. For example: 3 x 5 = 15 and 5x3 = 15

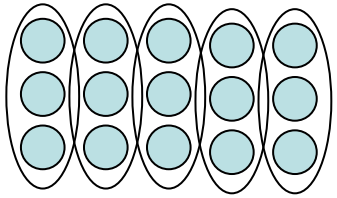
The third stage in written multiplication

Children will continue to use:

Children should be able to model a multiplication calculation using an array.



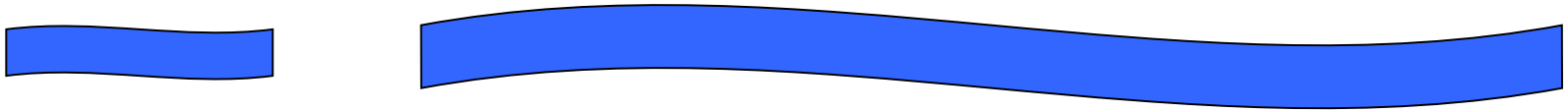
3 x 5 = 15



5 x 3 = 15

Scaling

e.g. Find a ribbon that is 4 times as long as the blue ribbon



5cm

20cm

Using symbols to stand for unknown numbers to complete equations using inverse operations

? x 5 = 20 3 x ? = 18 ? x ? = 32

The fourth stage in written multiplication

Partitioning

38 x 5 = (30 x 5) + (8 x 5)

= 150 + 40

= 190

Grid method

TU x U
(Short multiplication – multiplication by a single digit)

23 x 8

Children will approximate first
23 x 8 is approximately 25 x 8 = 200

x	20	3
8	160	24

160
+24
184

Vocabulary

Divide, dividing, division

Number sentence

Calculation

Repeated subtraction

Number line

Sharing

Grouping

Remainders

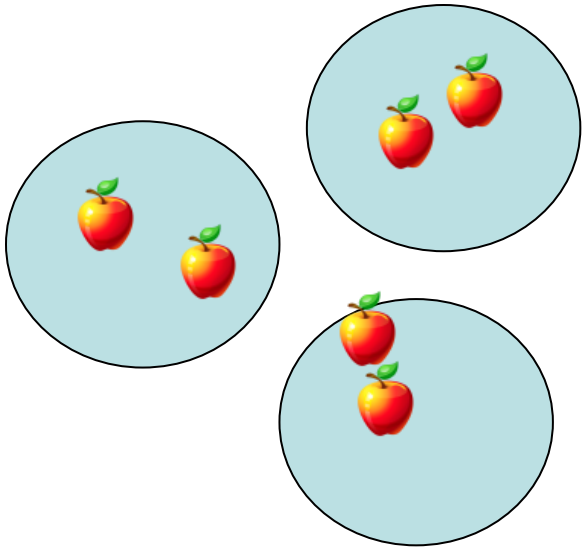
Chunking

multiple

Division

The first stage in written division

Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.

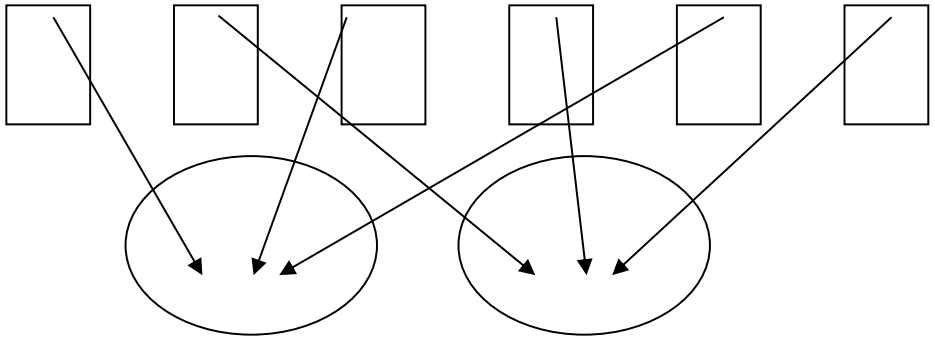


The second stage in written division

Children will develop their understanding of division and use jottings to support calculation.

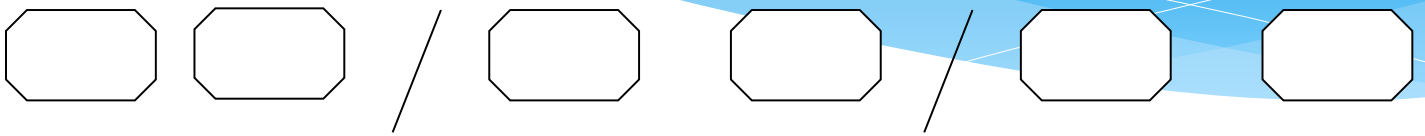
Sharing equally

6 sweets shared between 2 people, how many do they each get?



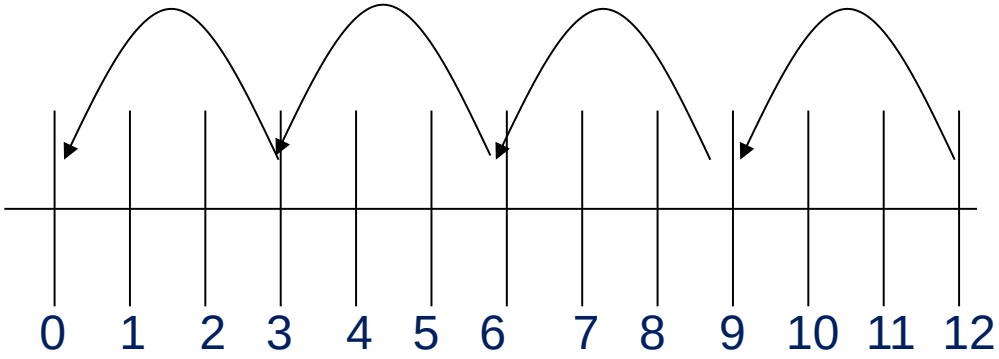
Grouping or repeated subtraction

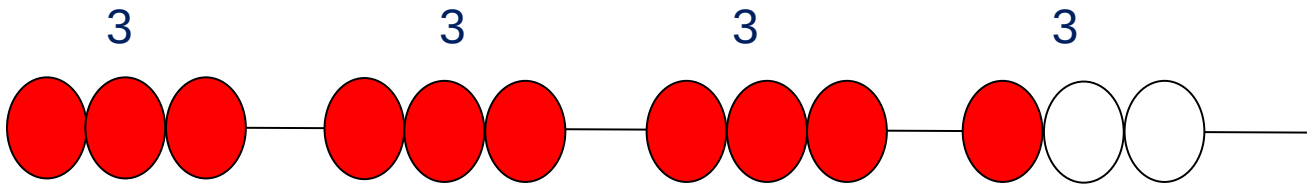
There are 6 sweets, how many people can have 2 sweets each?



Repeated subtraction using a number line or bead bar

12 ÷ 3 = 4





The third stage in written division

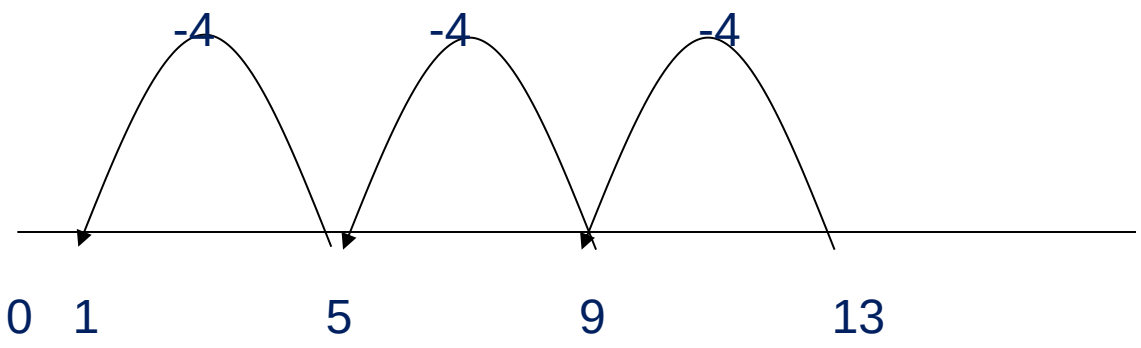
The emphasis is now on grouping rather than sharing.

Repeated subtraction using a number line

Children will use an empty number line to support their calculation.

Children should also move onto calculations involving remainders.

13 ÷ 4 = 3 r 1



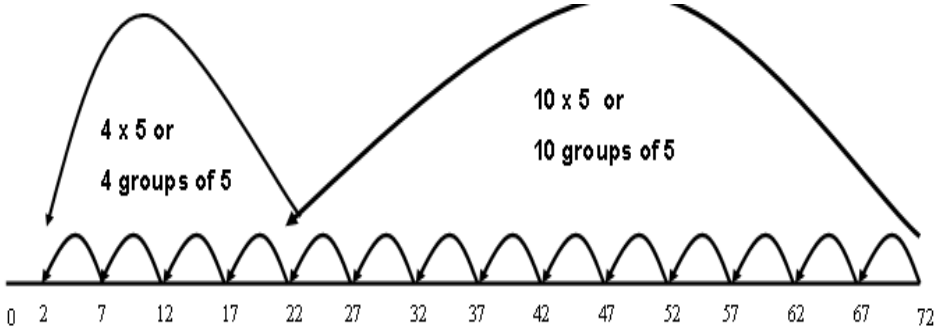
The fourth stage in written division

Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor (the number dividing by). Initially, these will be multiples of 10s, 5s, 2s and 1s – numbers with which the children are more familiar.

TU ÷ U

72 ÷ 5 = 14 r 2

Use of number line:



Then onto the vertical method known as 'chunking':

72 ÷ 5 lies between 50 ÷ 5 = 10 and 100 ÷ 5 = 20

72
-50 (10 groups) or (10 x 5)
22
-20 (4 groups) or (4 x 5)
2

Answer : 14 remainder 2

Addition and subtraction

789 + 642 becomes

$$\begin{array}{r} 7\ 8\ 9 \\ +\ 6\ 4\ 2 \\ \hline 1\ 4\ 3\ 1 \\ \hline \quad 1\ \ 1 \end{array}$$

Answer: 1431

874 – 523 becomes

$$\begin{array}{r} 8\ 7\ 4 \\ -\ 5\ 2\ 3 \\ \hline 3\ 5\ 1 \\ \hline \end{array}$$

Answer: 351

932 – 457 becomes

$$\begin{array}{r} \overset{8}{\not{9}}\ \overset{12}{\not{3}}\ \overset{1}{2} \\ -\ 4\ 5\ 7 \\ \hline 4\ 7\ 5 \\ \hline \end{array}$$

Answer: 475

932 – 457 becomes

$$\begin{array}{r} \quad 1\ \ 1 \\ 9\ 3\ 2 \\ -\ \not{4}\ \not{5}\ 7 \\ \quad 5\ \ 6 \\ \hline 4\ 7\ 5 \\ \hline \end{array}$$

Answer: 475

Short multiplication

24 × 6 becomes

$$\begin{array}{r} {\scriptstyle 2}\ \ 4 \\ \times\ \ 6 \\ \hline 1\ 4\ 4 \\ \hline {\scriptstyle 2} \end{array}$$

Answer: 144

342 × 7 becomes

$$\begin{array}{r} 3\ 4\ 2 \\ \times\ \ 7 \\ \hline 2\ 3\ 9\ 4 \\ \hline {\scriptstyle 2}\ {\scriptstyle 1} \end{array}$$

Answer: 2394

2741 × 6 becomes

$$\begin{array}{r} 2\ 7\ 4\ 1 \\ \times\ \ \ \ 6 \\ \hline 1\ 6\ 4\ 4\ 6 \\ \hline {\scriptstyle 4}\ {\scriptstyle 2} \end{array}$$

Answer: 16 446

Long multiplication

24 × 16 becomes

$$\begin{array}{r} {\scriptstyle 2} \\ 2\ 4 \\ \times\ 1\ 6 \\ \hline 2\ 4\ 0 \\ 1\ 4\ 4 \\ \hline 3\ 8\ 4 \\ \hline \end{array}$$

Answer: 384

124 × 26 becomes

$$\begin{array}{r} {\scriptstyle 1}\ {\scriptstyle 2} \\ 1\ 2\ 4 \\ \times\ \ 2\ 6 \\ \hline 2\ 4\ 8\ 0 \\ 7\ 4\ 4 \\ \hline 3\ 2\ 2\ 4 \\ \hline {\scriptstyle 1}\ {\scriptstyle 1} \end{array}$$

Answer: 3224

124 × 26 becomes

$$\begin{array}{r} {\scriptstyle 1}\ {\scriptstyle 2} \\ 1\ 2\ 4 \\ \times\ \ 2\ 6 \\ \hline 7\ 4\ 4 \\ 2\ 4\ 8\ 0 \\ \hline 3\ 2\ 2\ 4 \\ \hline {\scriptstyle 1}\ {\scriptstyle 1} \end{array}$$

Answer: 3224

Short division

98 ÷ 7 becomes

$$\begin{array}{r} 1\ 4 \\ 7\overline{)\,9\ 8} \\ {\scriptstyle 2} \end{array}$$

Answer: 14

432 ÷ 5 becomes

$$\begin{array}{r} 8\ 6\ \text{r}\,2 \\ 5\overline{)\,4\ 3\ 2} \\ {\scriptstyle 3} \end{array}$$

Answer: 86 remainder 2

496 ÷ 11 becomes

$$\begin{array}{r} 4\ 5\ \text{r}\,1 \\ 11\overline{)\,4\ 9\ 6} \\ {\scriptstyle 5} \end{array}$$

Answer: $45\frac{1}{11}$

Long division

432 ÷ 15 becomes

$$\begin{array}{r} 2\ 8\ \text{r}\,12 \\ 15\overline{)\,4\ 3\ 2} \\ 3\ 0\ 0 \\ \hline 1\ 3\ 2 \\ 1\ 2\ 0 \\ \hline 1\ 2 \end{array}$$

Answer: 28 remainder 12

432 ÷ 15 becomes

$$\begin{array}{r} 2\ 8 \\ 15\overline{)\,4\ 3\ 2} \\ 3\ 0\ 0 \quad 15\times 20 \\ \hline 1\ 3\ 2 \\ 1\ 2\ 0 \quad 15\times 8 \\ \hline 1\ 2 \end{array}$$

$\frac{\cancel{12}}{\cancel{15}} = \frac{4}{5}$

Answer: $28\frac{4}{5}$

432 ÷ 15 becomes

$$\begin{array}{r} 2\ 8\cdot 8 \\ 15\overline{)\,4\ 3\ 2\cdot 0} \\ 3\ 0 \\ \hline 1\ 3\ 2 \\ 1\ 2\ 0 \\ \hline 1\ 2\ 0 \\ 1\ 2\ 0 \\ \hline 0 \end{array}$$

Answer: 28·8